Least-squares Kirchhoff migration using traveltimes based on the maximum amplitude criterion by the rapid expansion method

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Abstract

In this paper, we developed a migration method based on the least squares method (LSM), which uses the Kirchhoff operator. The Kirchhoff operator is currently the most widely used migration operator and it is generally based on ray tracing algorithms which often does not work well in complex media. To improve the Kirchhoff operator performance for modeling and migration and use it in an effective way with our LSM procedure, the traveltimes are computed using a maximum amplitude criterion by the rapid expansion method (REM). Using Kirchhoff operators with this alternative way to compute traveltimes we implemented an optimization procedure seeking a correct reconstruction of the reflectors in order to obtain subsurface images with better seismic resolution. The proposed least-square Kirchhoff migration with REM (LSKM-REM) was tested on the Marmousi dataset and the results obtained with the proposed optimized procedure showed higher quality images when compared with the conventional Kirchhoff migration results.

Introduction

The Kirchhoff migration has been commonly used in seismic processing due to low computational cost, but the migration and modeling using the Kirchhoff operator is based on ray tracing which often does not provide a good image. Thereby, it becomes increasingly necessary to use other ways to get the traveltimes. Therewith, we implemented an optimization method with the Kirchhoff operator with traveltimes based on the REM (Kosloff, 1989) to improve the quality of the final migrated image. The LSKM-REM method implemented uses arrival traveltimes obtained by the REM using the maximum amplitude criterion. The LSKM-REM consists of finding a model closer to true reflectivity. Thereby, for each model obtained iteratively, a new synthetic dataset is modeled and compared with the observed dataset. The final reflectivity model will be the one that best minimizes the objective function.

In 2014, Santos and Pestana proposed an inversion migration procedure based on least-squares Kirchhoff migration (LSKM) and it was tested with complex data. Their study shows that the LSKM scheme can noticeably reduce migration artifacts and improve seismic resolution. Additionally, the adapted gradient method was used in order to reduce computational cost and provide a better image with much higher quality (approximates the true reflectivity) minimizing the error between observations and predictions (Santos and Pestana, 2014).

In this work, our goal is to show that the LSKM with traveltimes tables obtained by the wave equation modeling procedure using the rapid expansion solution method and based on the maximum amplitude criterion can provide better optimized image with much higher quality when compared with the LSKM results using the ray tracing traveltimes. Therefore, the traveltimes in complex media using the REM with the maximum amplitude criterion can produce better results when applied combined with the Kirchhoff operator through an optimization procedure.

Theory

Assume that the linear forward modeling operator $\mathbf{L}$ satisfies (Schuster et al., 1993; Nemeth et al., 1999; Dai et al., 2012; Santos et al; 2014):

$$\mathbf{d} = \mathbf{L}\mathbf{m}, \tag{1}$$

where $\mathbf{d}$ is a matrix of modeled data and $\mathbf{m}$ is the reflectivity model matrix. The observed data $\mathbf{d}_o$ is described by:

$$\mathbf{d}_o = \mathbf{L}_o\mathbf{m}_o, \tag{2}$$

where $\mathbf{m}_o$ is the true earth reflectivity model matrix and $\mathbf{L}_o$ is the forward modeling operator for the actual model. Unless stated otherwise, we assume that $\mathbf{L} = \mathbf{L}_o$.

Seismic migration uses the transpose of the forward modeling (1) that is:

$$\mathbf{m}_{mig} = \mathbf{L}^T\mathbf{d}, \tag{3}$$

where $\mathbf{m}_{mig}$ is the migrated section. Substituting equation (1) in equation (3) yields:

$$\mathbf{m}_{mig} = \mathbf{L}^T\mathbf{L}\mathbf{m}. \tag{4}$$

The matrix $\mathbf{L}^T\mathbf{L}$ is Hessian matrix and defines $\mathbf{m}_{mig}$ as an $\mathbf{L}^T\mathbf{L}$ - filtered version of $\mathbf{m}$. The migration operator $\mathbf{L}^T$ will correctly reconstruct the actual earth model if $\mathbf{L}^T\mathbf{L}$ is the identity matrix $\mathbf{I}$. In most cases $\mathbf{L}^T\mathbf{L}$ is different from the identity matrix (Nemeth et al, 1999).

To obtain a better reflectivity image, the imaging problem can be represented as a least-squares inversion problem. The solution is obtained by minimizing the objective function $O(\mathbf{m})$, which is defined as the least-squares difference between the forward modeled data and recorded data $\mathbf{d}_o$:

$$O(\mathbf{m}) = ||\mathbf{L}\mathbf{m} - \mathbf{d}_o||^2 \tag{5}$$

The iterative solution reads:

$$\mathbf{G}_k = \mathbf{L}^T[\mathbf{Lm} - \mathbf{d}_o] \tag{6}$$

$$\alpha_k = \frac{\mathbf{G}_k^T \mathbf{G}_k}{(\mathbf{E}_k)^T(\mathbf{E}_k)} \tag{7}$$

$$\mathbf{m}_{k+1} = \mathbf{m}_k - \alpha_k \mathbf{G}_k. \tag{8}$$

Equation (6) represents the migration of the difference between modeled data and observed data associated with the model $\mathbf{m}$. This vector of migrated residue is in the same direction of the gradient vector defined in $\mathbf{m}$, but in opposite signal. The gradient vector is represented mathematically by $\mathbf{G}_k$, pointing in the direction of maximum slope (Santos et al., 2013). α_k is a scale factor and called the step size which is computed in each k-iteration. Equation (8) is used to update the reflectivity model through this optimized procedure in each iteration.

The Kirchhoff operator

This paper considers least squares prestack depth migration using a Kirchhoff operator. Kirchhoff migration involves integrating traces amplitudes over a reflectivity model. After traveltimes and Kirchhoff weights have been calculated, the migration process can be written as a trace by trace process.

In case of LSM the forward and adjoint operator are required. Given the simplicity of the Kirchhoff adjoint operator, the forward operator is straightforward to define.

The forward Kirchhoff operator can be written as

$$d(s,g,t) = \sum_{N_x}\sum_{N_z} m(x,z)\, K(s,g,x,z,t) \tag{9}$$

where $d(s,g,t)$ is the data, $m(x,z)$ is the model, and $K(s,g,x,z,t)$ are the Kirchhoff weights. Here t is the traveltime that is normally obtained via ray tracing through the velocity model.

The adjoint, or migration operator, can be written as

$$m(x,z) = \sum_{N_s * N_g} d(s,g,t)\, K(s,g,x,z,t) \tag{10}$$

Given the forward Kirchhoff operator, L, as defined by equation (1), data can be generated from the reflectivity model. Given recorded data, we may want to collapse diffractions to the position where reflections occurred. To do this we should use the inverse of the forward operator, L^{-1}. Typically for imaging the inverse is approximated by the adjoint operator, L^T, equation (3).

Travel time by maximum amplitude

Computation of the traveltimes is the heart of the Kirchhoff algorithm. Ray tracing is the most used method to compute the arrival times. An alternative procedure for computing the arrival times for a grid of points is by solving the eikonal equation by finite-differences method.

Here we suggest an alternative procedure to calculate the arrival times based on the maximum amplitude criterion. It is based on the wave equation solution using the rapid expansion method (REM), first presented in Kosloff et al. (1989). To determine the traveltimes, we use the maximum amplitude criterion to identify time for the direct wave computed from the modeling. This maximum amplitude criterion is justified since the direct wave has the maximum amplitude at the direct arrival time. Late arrivals have smaller amplitudes due to the transmission losses. Using the maximum amplitude criterion the travel time $T_{i,j}$ for each grid point (i, j) at each time step k of the Chebyshev recursion is updated and after finishing the last time step of modeling the traveltimes table is saved in a file to be used as input for the Kirchhoff migration and modeling procedures.

Results

To validate the LSKM implemented here, we have used the 2-D Marmousi dataset which is a very complex depth model. The true velocity model is presented in Figure 1, a smoothed version of the velocity model is shown in Figure 2 and the reflectivity model for the Marmousi model is shown in Figure 3. First, we migrated the original Marmousi dataset that was generated by a finite-difference scheme. Using the Kirchhoff method (KM) with traveltime computed by a ray tracing method (rayt2 code from the Seismic Unix - CWP) and using the smoothed version of the velocity model (Figure 2) we obtained the migrated result shown in Figure 4. The result using the KM-REM is shown in Figure 5. From these results we can notice that the migration results obtained with both method are reasonable. However, when the LSKM and LSKM-REM optimization procedures were applied, Figures 6 and 7, with LSKM, and Figures 8 and 9, with LSKM-REM, for 10 and 50 iteration, respectively, we did not observe an increase of resolution as expected. We can not notice a better result with the LSKM-REM in comparison with the LSKM. This method using an optimization procedure could better image the reflectors throughout the model, improving the resolution of structures in which the first migration result did not properly image.

From these results, we can notice that both optimization procedures using the Kirchhoff operator did not improve effectively the migration results of the first migration. This can be attributed to some issues as the wavelet in the observed dataset (FD Marmousi dataset) and the modeled dataset used on the LSKM have different wavelet and some issues related with the amplitude of both dataset. To obtain better results we need a good estimation of the wavelet and also a good amplitude relation between the datasets.

To prove that the proposed method can produce high resolution results, we generate a new dataset applying the REM traveltimes and using a SU demigration code. The obtained dataset was denoted as our observed dataset. This new dataset is generated knowing the source wavelet and during the optimization procedure, on each iteration, the modeled dataset (calculated dataset) will have the same wavelet and both of them will have a more reliable comparison in terms of amplitude.

After obtaining the first migration result using the KM-REM, which is shown in Figure 10, we continue the optimization procedure and using the LSKM-REM, we can notice from Figures 11 and 12, results using 10 and 50 iteration, respectively, that after these iterations the results provide good images with much higher resolution in comparison with the first migration result (Figure 10). To prove its efficacy we also present the objective function (Figure 13) which tends to a minimum, as expected.

Conclusions

In this paper we have proposed a least-squares Kirchhoff migration method using traveltime arrivals obtained through the rapid expansion method, which is a wave equation solution method, using the maximum amplitude criterion. From our tests, the LSM results for the Marmousi modeled dataset using the Kirchhoff-REM provided images with higher resolution than the ones obtained with the original data. We also noticed that the LSKM-REM showed results with a much better lateral resolution of the reflectivity model without requiring a costly acquisition of denser dataset. Additionally we also concluded that the least-squares Kirchhoff migration combined with the rapid expansion method helped to reduce artifacts in a natural way by generating the reflectivity model that predicts the observed data in a least-squares sense.

The present version of LSKM-REM uses an adapted gradient method to improve the convergence of the method and provide migrated images with higher resolution. From the imaging results, we can see that for few iterations, such as 10, the LSKM-REM shows a better imaging for the modeled data using the Kirchhoff-REM method than the original data. We can also see from the graph of the residual error versus the iteration number that the residual error gradually decreases as the iteration number increases, as expected. Further works is needed to apply this method for synthetic and real datasets. We need to have a good estimation of the source in order to obtain a reliable amplitude comparison between the modeled and observed datasets. We still need to improve the convergence speed up of the proposed method by using other inversion methods and also using different objective functions.

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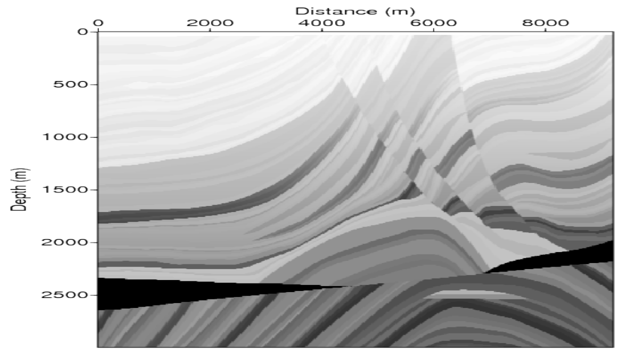

Figure 1: Velocity field of Marmousi model.

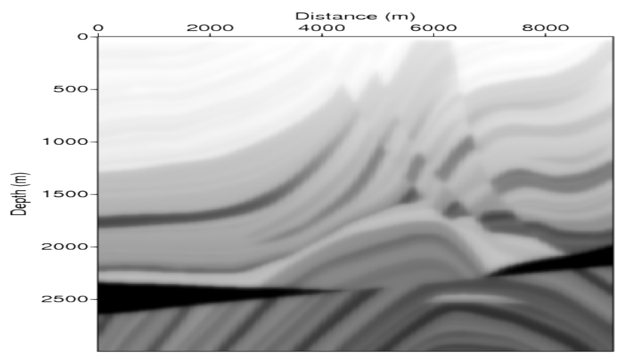

Figure 2: Smoothed version of the Marmousi velocity field.

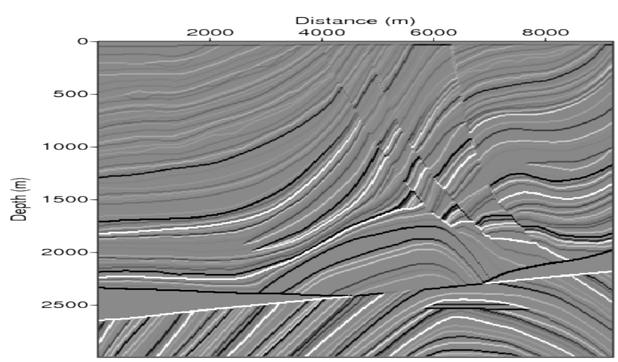

Figure 3: Reflectivity of the Marmousi model.

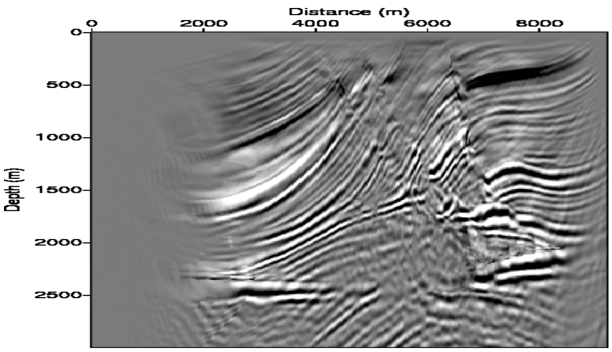

Figure 4: Kirchhoff migration result of original Marmousi dataset (FD modeled dataset) using traveltimes tables computed by REM.

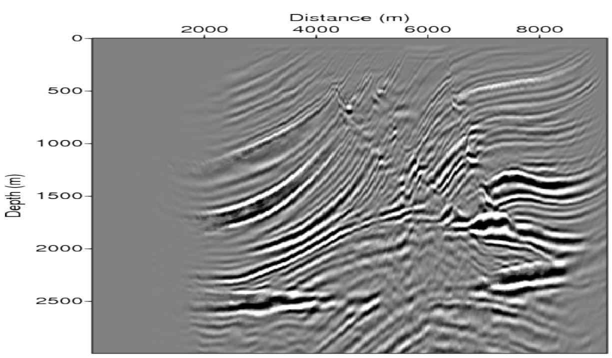

Figure 5: Kirchhoff migration result of the original Marmousi dataset using traveltime tables computed by ray-tracing (Seismic-Unix code) using the smoothed velocity model shown in Figure 2.

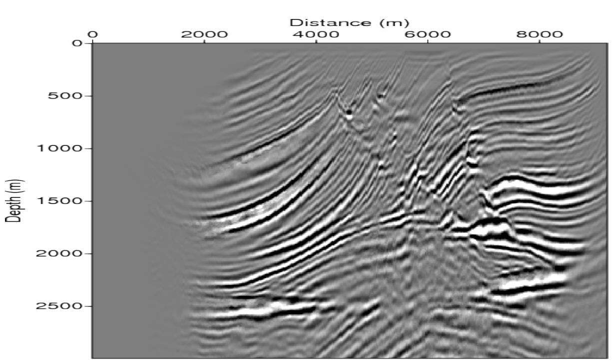

Figure 6: Least-squares Kirchhoff migration result of the original Marmousi dataset, after 10 iterations, using ray-tracing traveltime tables.

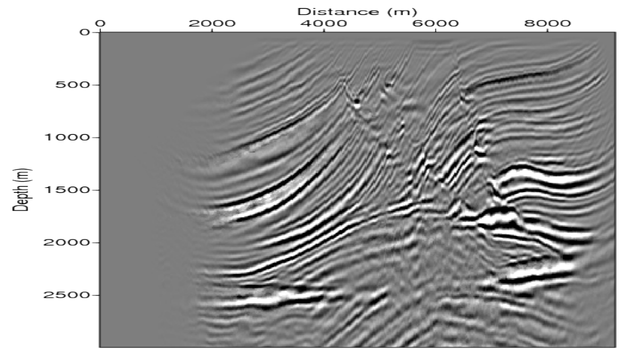

Figure 7: Least-squares Kirchhoff migration result of the original Marmousi dataset, after 50 iterations, using ray-tracing traveltime tables.

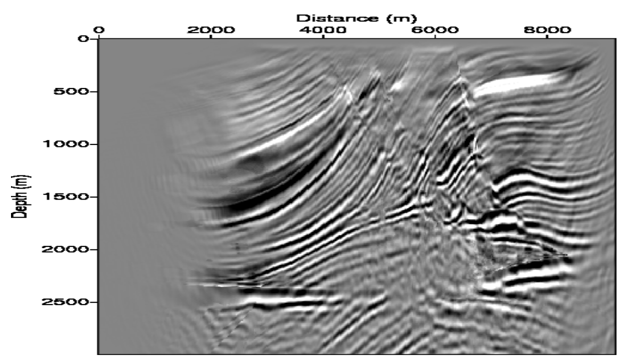

Figure 8: Least-square Kirchhoff migration result of the original Marmousi dataset, after 10 iterations, using REM traveltime tables.

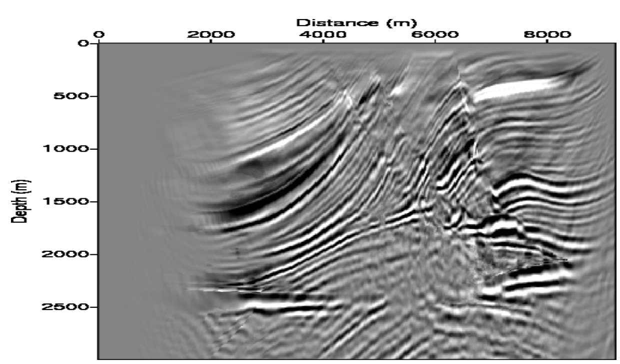

Figure 9: Least-square Kirchhoff migration result of the original Marmousi dataset, after 10 iterations, using ray-tracing traveltime tables.

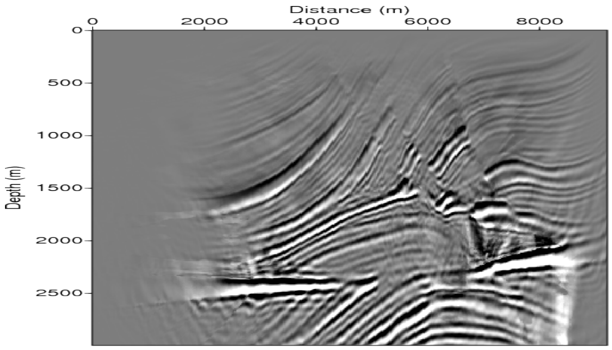

Figure 10: Kirchhoff migration result of the modelled Marmousi dataset using REM traveltime tables.

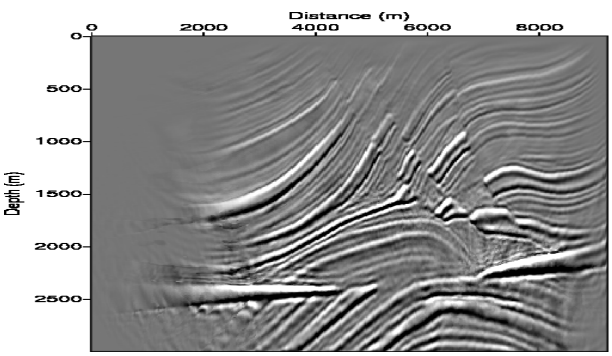

Figure 11: Least-squares Kirchhoff migration result of the modeled Marmousi dataset, after 10 iterations, using REM traveltime tables.

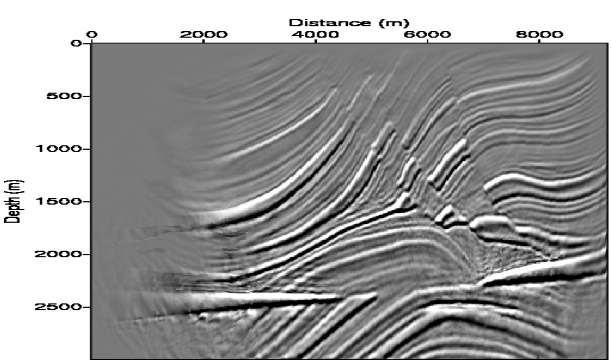

Figure 12: Least-squares Kirchhoff migration result of the modeled Marmousi dataset, after 50 iterations, using REM traveltime tables.

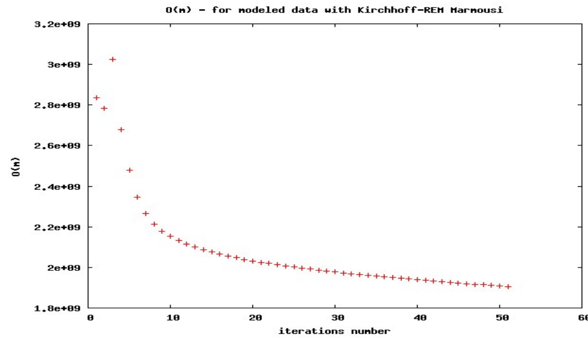

Figure 13: Data residual versus iteration number for the modeled dataset by Kirchhoff with REM's traveltimes tables.